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Supersymmetry breaking, $\mathcal{M}$-theory and fluxes

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Abstract: We consider warped compactifications of $\mathcal{M}$-theory to three-dimensional Minkowski space on compact eight-manifolds. Taking all the leading quantum gravity corrections of eleven-dimensional supergravity into account we obtain the solution to the equations of motion and Bianchi identities. Generically these vacua are not supersymmetric and yet have a vanishing three-dimensional cosmological constant.

Keywords: M-Theory, Superstring Vacua.

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Contents

1. Introduction

For a long time it has been known that string theory suffers from the vacuum selection problem [1, 2]. Different shapes and sizes of the compactified dimensions lead to many physically inequivalent degenerate vacua. This is not a very attractive situation. Definite predictions for all the dimensionless constants of nature can only be made if string theory has a unique vacuum state. Thus over the years different mechanisms have been developed to address this situation[1] but no clear progress had been made so far. Gukov, Vafa and Witten [4] realized[2] that if we consider a warped compactification of $\mathcal{M}$-theory on eight-manifolds with non-vanishing fluxes for tensor fields[3] the expectation values for the complex structure and Kähler structure moduli fields are no longer arbitrary. Most of them are fixed in terms of the discrete fluxes found in [12, 13]. As shown by Giddings, Kachru and Polchinski [14] a similar situation appears in the type-IIB theory.

In this paper we will be interested in finding all vacua for warped compactifications of $\mathcal{M}$-theory on compact eight-manifolds. Compact manifolds are of special interest as they lead to a finite three-dimensional Planck scale. Taking all the leading quantum gravity corrections of $\mathcal{M}$-theory into account it is our goal to derive the most general solution to the equations of motion for compactifications on eight-dimensional Kähler manifolds to three-dimensional Minkowski space. This solution

[1]For a review see e.g. [3].
[2]See also [5].
[3]Recent work on theories which include non-vanishing fluxes was done in [6]–[11].

will be written in terms of first order constraints which will be much easier to solve than the second order constraints coming from the equations of motion. We hope this will be useful in order to construct new interesting concrete models in the future. Generically we will find solutions which have a vanishing three-dimensional cosmological constant and broken supersymmetry. Such an interesting situation has appeared recently in the no-scale models of [14].

In section 2 we will derive the solution to the equations of motion and Bianchi identities for compactifications of $\mathcal{M}$-theory to three-dimensional Minkowski space. In section 3 we will summarize our solution. In section 4 we discuss the constraints imposed by supersymmetry and the possibility to break supersymmetry to $N = 0$ by turning on some specific fluxes. In section 5 we will review the interpretation of the flux constraints that our solution obeys and the relation to the moduli space problem of $\mathcal{M}$-theory compactifications. Many of the moduli fields can be stabilized once the constraints are taken into account. We will finish in section 6 with some concluding remarks.

2. Solution to the equations of motion

The bosonic part of the action of eleven-dimensional supergravity [15] including the leading quantum corrections [16]–[19] has the following form

$$\begin{aligned} S &= S_0 + S_1\,, \\ S_0 &= -\frac{1}{2\kappa^2}\int d^{11}x\sqrt{-g}\left[R - \frac{1}{2\cdot 4!}F^2 - \frac{1}{6\cdot 3!\cdot (4!)^2}\epsilon_{11}CFF\right], \\ S_1 &= -b_1 T_2 \int d^{11}x\sqrt{-g}\left(J_0 - \frac{1}{2}E_8\right) + T_2\int C\wedge X_8\,. \end{aligned} \tag{2.1}$$

Here $b_1 = \frac{1}{(2\pi)^4 3^2 2^{13}}$ and T_2 is the membrane tension related to the eleven-dimensional Newton's constant by

$$T_2 = \left(\frac{2\pi^2}{\kappa^2}\right)^{1/3}. \tag{2.2}$$

We will be using the conventions of [19]. Furthermore, $F = dC$ is the four-form field strength and J_0, E_8 and X_8 are quartic polynomials in the eleven-dimensional Riemann tensor. The explicit form of the polynomial J_0 is

$$\begin{aligned} J_0 = 3\cdot 2^8\Big(&R^{HMNK}R_{PMNQ}R_H{}^{RSP}R^Q{}_{RSK} + \frac{1}{2}R^{HKMN}R_{PQMN}R_H{}^{RSP}R^Q{}_{RSK}\Big) + \\ &+ O(R_{MN})\,. \end{aligned} \tag{2.3}$$

The polynomial E_8 is an eleven-dimensional generalization of the eight-dimensional Euler integrant and is given by

$$E_8 = \frac{1}{3!}\epsilon^{ABCM_1N_1\cdots M_4N_4}\epsilon_{ABCM'_1N'_1\cdots M'_4N'_4}R^{M'_1N'_1}{}_{M_1N_1}\cdots R^{M'_4N'_4}{}_{M_4N_4}\,. \tag{2.4}$$

Capital letters range over $0, \ldots, 10$. The expression for X_8 is

$$X_8 = \frac{1}{192(2\pi)^4}\left[\mathrm{tr}\, R^4 - \frac{1}{4}(\mathrm{tr}\, R^2)^2\right]. \tag{2.5}$$

The Einstein equation which follows from this action is

$$R_{MN} - \frac{1}{2}g_{MN}R - \frac{1}{12}T_{MN} = -\beta\frac{1}{\sqrt{-g}}\frac{\delta}{\delta g^{MN}}\left(\sqrt{-g}\left(J_0 - \frac{1}{2}E_8\right)\right), \tag{2.6}$$

where T_{MN} is the energy momentum tensor of F given by

$$T_{MN} = F_{MPQR}F_N{}^{PQR} - \frac{1}{8}g_{MN}F^2_{PQRS}, \tag{2.7}$$

and we have set $\beta = 2\kappa^2 b_1 T_2$.

Without sources the field strength obeys the Bianchi identity

$$dF = 0, \tag{2.8}$$

and the equation of motion

$$d * F = \frac{1}{2}F \wedge F + \frac{\beta}{b_1}X_8. \tag{2.9}$$

In the following we shall be interested in considering compactifications on eight-manifolds. Our goal is to derive the general conditions under which the equations of motion have a solution by a perturbation expansion in t, where t is the radius of the eight-manifold which is taken to be large. Such a large radius expansion was used in [20] for compactifications of the heterotic string. We consider the background metric to be a warped product [12]

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2B(y)}g_{mn}dy^m dy^n, \tag{2.10}$$

where $\eta_{\mu\nu}$ describes the three-dimensional Minkowski space M_3. The metric g_{mn} is taken to be of order t^2

$$g_{mn} = t^2 g^{(0)}_{mn} + g^{(1)}_{mn} \cdots, \tag{2.11}$$

and describes the eight-dimensional internal manifold Y_4. In our notation the indices $m, n, \ldots$ are real. In this paper we will be interested in compactifications where Y_4 is Kähler. It would be interesting to find the generalization of our analysis to non-Kähler manifolds such as the Spin(7) holonomy manifolds considered in [21]. To derive the three-dimensional equations of motion by a perturbative expansion we need to analyze the scaling behaviors of all fields as function of the radius. From (2.11) it follows that the inverse metric scales as $g^{mn} \sim 1/t^2$, the Riemann tensor scales with t^2 and the scalar curvature is of order t^{-2}

$$R = g^{mn}R_{mn} = g^{mn}g^{kl}R_{mknl} = t^{-2}R^{(0)} + t^{-8}R^{(1)} + \cdots. \tag{2.12}$$

Furthermore, the Ricci tensor is of order zero

$$R_{mn} = R^{(0)}_{mn} + t^{-6} R^{(1)}_{mn} + \cdots , \tag{2.13}$$

while from (2.4) we find that the quartic polynomial of the Riemann tensor scales as

$$E_8(Y_4) = t^{-8} {E_8}^{(0)}(Y_4) + \cdots , \tag{2.14}$$

and a similar expansion for J_0. To leading order in the large t-expansion one can replace the Riemann tensor appearing in J_0 (2.3) by the Weyl tensor. This will be useful later on.

In compactifications with maximally symmetric three-dimensional space-time the field strength is a sum

$$F = F_1 + F_2 , \tag{2.15}$$

where F_1 has the form

$$F_{\mu\nu\rho m} = \epsilon_{\mu\nu\rho} \partial_m f , \tag{2.16}$$

with indices on the three-dimensional Minkowski space while F_2 has only indices on the eight-manifold. Here $f = f(y)$ is a function of the internal coordinates that will be determined later on. This form of the field strength follows from Poincaré invariance. The above ansatz for F_1 satisfies the Bianchi identity for the external component of the tensor field.

In order to derive the field equations order by order in the t-expansion we will make the following ansatz for the scaling behavior of the tensor fields

$$f = f^{(0)} + t^{-6} f^{(1)} + \cdots \tag{2.17}$$

and

$$F_2 = {F_2}^{(0)} + t^{-6} {F_2}^{(1)} + \cdots . \tag{2.18}$$

From (2.16) and (2.17) we see that F_1 has a similar expansion as F_2. Also, we will be making an ansatz for the scaling behavior of the warp factors

$$e^X = 1 + \frac{X^{(1)}}{t^6} + \cdots , \tag{2.19}$$

with $X = A, B$.

Combining the leading orders of the external and internal Einstein's equations we see that the internal manifold is Ricci flat

$$R^{(0)}_{mn} = 0 . \tag{2.20}$$

Also, the external component of the flux vanishes to leading order because $f^{(0)} =$ const.

To order t^{-8} the external component of Einstein's equation is

$$-4\Box A^{(1)} - 14\Box B^{(1)} - \frac{1}{48}({F_2}^{(0)})^2 + R^{(1)} + \frac{\beta}{2}{E_8}^{(0)}(Y_4) = 0\,. \tag{2.21}$$

Here we used the fact that we can neglect the contribution of the warp factor to the (Riemann)^4 terms. Thus the right-hand side of (2.6) can be evaluated on a product space of the form $M_3 \times Y_4$. To obtain the contribution coming from E_8 we have taken into account that for these product spaces we have [22]

$$E_8(M_3 \times Y_4) = -E_8(Y_4) - 8R(M_3)E_6(Y_4)\,. \tag{2.22}$$

Here $E_6(Y_4)$ is the cubic polynomial of the internal Riemann tensor

$$E_6(Y_4) = 2^8 (R_a{}^b{}_c{}^d R_d{}^e{}_b{}^g R_e{}^a{}_g{}^c + R_a{}^c{}_b{}^d R_d{}^b{}_e{}^g R_g{}^e{}_c{}^a)\,. \tag{2.23}$$

At this point we will be assuming that the internal manifold is Kähler so that we can introduce complex coordinates which we will be denoting by $a, b, \bar{a}, \bar{b}, \ldots$. Since $R(M_3)$ is the scalar curvature of the external space the second term in the previous equation actually vanishes. To evaluate the contribution from J_0 to the external Einstein equation we have used the fact that J_0 is the sum of an external and an internal part. The external part vanishes because the Weyl tensor vanishes identically in three dimensions [23]. The internal part does not contribute because it vanishes for Ricci flat Kähler manifolds. This can be easily checked using the explicit expression for J_0 appearing in (2.3).

We now would like to consider the order t^{-6} of the internal Einstein equation. Let us start with the $(a, \bar{b})$ component which takes the form

$$R^{(1)}_{a\bar{b}} - \frac{1}{2}g^{(0)}_{a\bar{b}}R^{(1)} - 3\partial_a\partial_{\bar{b}}C^{(1)} + 3g^{(0)}_{a\bar{b}}\Box C^{(1)} - \frac{1}{12}T^{(1)}_{a\bar{b}} = \beta\partial_a\partial_{\bar{b}}E_6(Y_4)\,. \tag{2.24}$$

Here we have introduced the notation $C^{(1)} = A^{(1)} + 2B^{(1)}$. To evaluate the right-hand side of (2.6) we used the identity

$$\frac{\delta}{\delta g^{a\bar{b}}}J_0 = -\partial_a\partial_{\bar{b}}E_6(Y_4)\,, \tag{2.25}$$

which is valid for Ricci flat Kähler manifolds. This can be checked using the results of [24, 25, 26] or by a lengthy but straigthforward calculation. There is one point with which one has to be careful though, which is the scheme dependence of J_0. The explicit form of the terms that involve the Ricci tensor in (2.3) can be changed using the equations of motion. This issue has been discussed in detail in the literature for the type-IIA higher order interactions. We have done the above calculation in the same scheme that was used in [24, 25, 26] or more concretely in [27].

Taking the trace of (2.24) with the metric $g^{(0)}_{a\bar{b}}$ we obtain an expression for the scalar curvature of the internal manifold

$$R^{(1)} = 7\Box A^{(1)} + 14\Box B^{(1)} - \frac{\beta}{3}\Box E_6(Y_4)\,. \tag{2.26}$$

Here we have used that the energy-momentum tensor is traceless in eight dimensions. Inserting this into the external Einstein equation (2.21) we obtain a determining equation for the warp factor $A^{(1)}$

$$3\Box A^{(1)} - \frac{1}{48}({F_2}^{(0)})^2 - \frac{\beta}{3}\Box E_6(Y_4) + \frac{\beta}{2}{E_8}^{(0)}(Y_4) = 0\,. \tag{2.27}$$

The F_1 equation of motion states

$$\Box f - \frac{1}{48}F_2^{(0)}\tilde{\star}F_2^{(0)} + \frac{\beta}{2}E_8^{(0)}(Y_4) = 0\,, \tag{2.28}$$

where by $\tilde{\star}$ we mean the Hodge dual with respect to the eight-dimensional metric. Substracting this from eq. (2.27) and integrating over the compact eight-manifold we obtain the condition that $F_2^{(0)}$ has to be self-dual

$$F_2^{(0)} = \tilde{\star}F_2^{(0)}\,. \tag{2.29}$$

Since $F_2^{(0)}$ is self-dual we can compare (2.27) with (2.28) to get a relation between the external component of the tensor field, the warp factor $A^{(1)}$ and the polynomial E_6

$$f^{(1)} = 3A^{(1)} - \frac{\beta}{3}E_6(Y_4) + \text{const}\,. \tag{2.30}$$

There is an integrability condition for being able to solve eqs. (2.27) and (2.28) for $A^{(1)}$ and f, respectively [20]. The source terms must be orthogonal to the zero modes of the operator $\Box$. The only zero modes of the operator $\Box$ on a compact manifold are constants, so that the integrability condition for both equations becomes

$$\int_{Y_4} F_2^{(0)} \wedge F_2^{(0)} + \frac{\chi}{12} = 0\,, \tag{2.31}$$

where χ is the Euler number of the eight-manifold. This condition has been found before in [12, 13]. It indicates that compactifications on eight-manifolds with non-vanishing Euler number are only consistent if fluxes are turned on.

Having shown the self-duality of $F_2^{(0)}$ let us go back to the internal Einstein equation (2.24). It turns out that any self-dual tensor in eight dimensions satisfies [28]

$$F_{mpqr}{F_n}^{pqr} = \frac{1}{8}g_{mn}F^2_{pqrs}\,. \tag{2.32}$$

Due to this identity the energy momentum tensor $T^{(1)}_{mn}$ vanishes identically, so that it does not contribute to the internal Einstein's equations. Equation (2.24) then becomes

$$R^{(1)}_{a\bar{b}} - \frac{1}{2}g^{(0)}_{a\bar{b}}\Box\left(C^{(1)} - \frac{\beta}{3}E_6(Y_4)\right) = 3\partial_a\partial_{\bar{b}}\left(C^{(1)} + \frac{\beta}{3}E_6(Y_4)\right)\,. \tag{2.33}$$

Recall that for a Kähler manifold the Ricci tensor and the metric are curl free. Taking the curl of (2.33) gives

$$\partial_a \Box \left(C^{(1)} - \frac{\beta}{3} E_6(Y_4) \right) = 0 \,. \tag{2.34}$$

For a compact eight-manifold the solution to this equation is

$$2B^{(1)} + A^{(1)} - \frac{\beta}{3} E_6(Y_4) = \text{const} \,, \tag{2.35}$$

This determines the warp factor $B^{(1)}$ in terms of $A^{(1)}$.

Furthermore we observe that to this order the internal manifold is no longer Ricci flat because the Ricci tensor satisfies

$$R^{(1)}_{a\bar{b}} = 2\beta \partial_a \partial_{\bar{b}} E_6(Y_4) \,. \tag{2.36}$$

This fact is familiar from the type-IIA theory in which the background metric is no longer Ricci flat to the next to leading order in the α' expansion once higher order interactions are taken into account [27]. This completes our discussion of the $(a, \bar{b})$ component of the internal Einstein equation.

The remaining Einstein equation takes the form

$$R^{(1)}_{ab} - 3\partial_a \partial_b C^{(1)} + \beta \partial_a \partial_b E_6(Y_4) = 0 \,, \tag{2.37}$$

and a similar expression for the antiholomorphic component. Here we have taken into account

$$\frac{\delta}{\delta g^{ab}} J_0 = \partial_a \partial_b E_6(Y_4) \,, \tag{2.38}$$

and the same result holds for the variation with respect to a metric with two anti-holomorphic indices. With the solution (2.35) for $C^{(1)}$ these equations become

$$R^{(1)}_{ab} = R^{(1)}_{\bar{a}\bar{b}} = 0 \,, \tag{2.39}$$

as it has to be for the metric to be Kähler.

It was shown in [27, 29, 30] that there always exists a Kähler metric on a Calabi-Yau manifold which satifies Einstein's equations (2.36) and (2.39) even if the manifold is no longer Ricci flat. We will be assuming that Y_4 is a Calabi-Yau manifold so that supersymmetry is not broken by the background metric but by the fluxes. It would be interesting to know if non-Kähler manifolds solve the next to leading order constraints.

Finally, the equation of motion for the internal component of the tensor field ${F_2}^{(0)}$ is

$$d(\tilde{\star} {F_2}^{(0)}) = 0 \,. \tag{2.40}$$

Since ${F_2}^{(0)}$ is closed and self-dual this equation is always satisfied and imposes no further constraints.

3. Summary of the solution to the equations of motion

The solution to the equations of motion and Bianchi identity for $\mathcal{M}$-theory compactified to three-dimensional Minkowki space on an eight-dimensional Kähler manifold is characterized by the following conditions.

- ▷ The field strength is of the form

$$F = F_1 + F_2 \,, \tag{3.1}$$

  where F_1 is the external component given by (2.16) and F_2 has only indices on the internal eight-manifold.

- ▷ To leading order the internal component of the field strength must be self-dual

$$\tilde{\star} {F_2}^{(0)} = {F_2}^{(0)} \,. \tag{3.2}$$

  and satisfy the integrability condition

$$\int_{Y_4} F_2^{(0)} \wedge F_2^{(0)} + \frac{\chi}{12} = 0 \,, \tag{3.3}$$

  where χ is the Euler number of the eight-manifold.

- ▷ The leading order the external component of the field strength vanishes

$${F_1}^{(0)} = 0 \,, \tag{3.4}$$

  while the next to leading order component ${F_1}^{(1)}$ is related to the warp factor $A^{(1)}$ by eq. (2.30).

- ▷ The warp factors $A^{(1)}$ and $B^{(1)}$ follow from eqs. (2.27) and (2.35).

- ▷ To leading order the internal manifold Y_4 is Ricci flat. To the next to leading order the internal manifold is no longer Ricci flat. The Ricci tensor is given by (2.36) and (2.39). These equations have a solution if Y_4 is a Calabi-Yau manifold.

Let us analyze the conditions under which the internal component of the field strength F_2 is self-dual [22]. The behavior under duality of a four-form on an eight-dimensional Kähler manifold is the following

$$\tilde{\star} f_{(4,0)} = f_{(4,0)} \,, \qquad \tilde{\star} f_{(3,1)} = -f_{(3,1)} \,, \qquad \tilde{\star} f_{(1,3)} = -f_{(1,3)} \,, \qquad \tilde{\star} f_{(0,4)} = f_{(0,4)} \,, \tag{3.5}$$

where in general $f_{(p,q)}$ denotes a form of type (p,q) with p holomorphic and q anti-holomorphic indices. In order to derive this result it is easiest to use the following representation of the epsilon tensor

$$\epsilon_{a\bar{b}c\bar{d}e\bar{f}g\bar{h}} = g_{a\bar{b}} g_{c\bar{d}} g_{e\bar{f}} g_{g\bar{h}} \pm \text{permutations} \,. \tag{3.6}$$

From (3.5) it follows that the self-duality constraint (3.2) imposes the conditions

$$F_{(1,3)} = F_{(3,1)} = 0 \,. \tag{3.7}$$

However, the constraint allows a $(2,2)$ form

$$F_{(2,2)} = f_{(2,2)} \,, \tag{3.8}$$

which is primitive

$$J \wedge f_{(2,2)} = 0 \,, \tag{3.9}$$

where J is the Kähler form of the manifold to leading order. This is so because every primitive $(2,2)$ form is self-dual. Of course, $f_{(2,2)}$ should be closed in order for the Bianchi identity to be satisfied. Notice that for an eight-manifold a self-dual $(2,2)$ form is not necessarily primitive. This situation is rather different than for threefolds where for $(2,1)$ forms primitivity and self-duality are equivalent. For a fourfold a self-dual $(2,2)$ form does not have to be primitive but a primitive $(2,2)$ form is self-dual. Finally, the constraint (3.2) allows a $(2,2)$ form which is not primitive [31]

$$F_{(2,2)} = J \wedge J f_{(0,0)} \,, \tag{3.10}$$

with $f_{(0,0)}$ closed.

Altogether, the equations of motion and Bianchi identities will be satisfied for ${F_2}^{(0)}$ of the form

$${F_2}^{(0)} = f_{(4,0)} + f_{(0,4)} + f_{(2,2)} + J \wedge J f_{(0,0)} \,. \tag{3.11}$$

This summarizes all the conditions describing the solution to the equations of motion and Bianchi identities. We now would like to compare with the constraints coming from supersymmetry.

4. Supersymmetry and supersymmetry breaking

The solution that we just presented does not need to be supersymmetric. Let us recall the constraints imposed by supersymmetry on these compactifications. In [12] it was shown that for a supersymmetric compactification of $\mathcal{M}$-theory on eight-manifolds the four-form is of type $(2,2)$, i.e.

$$F_{(4,0)} = F_{(0,4)} = F_{(3,1)} = F_{(1,3)} = 0 \,. \tag{4.1}$$

Further the non-vanishing component of F has to be primitive

$$F_{(2,2)} \wedge J = 0 \,. \tag{4.2}$$

Therefore, supersymmetry only allows a four-form flux that takes the form

$${F_2}^{(0)} = f_{(2,2)} \,. \tag{4.3}$$

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Comparing with the result coming from the equations of motion (3.11) we see that there is the interesting possibility that supersymmetry can be broken by turning on the $(4,0)$ form (or the corresponding $(0,4)$ form)

$$f_{(4,0)} \neq 0\,, \tag{4.4}$$

or a $(2,2)$ form that is not primitive. From (3.11) we see that such a non-primitive $(2,2)$ form is

$$F_{(2,2)} = J \wedge J f_{(0,0)}\,. \tag{4.5}$$

In both cases we know from the results of this paper that even if supersymmetry is broken after turning on these fluxes the three-dimensional cosmological constant vanishes. Such an interesting scenario was first discussed in the context of supersymmetry breaking by gluino condensation in the heterotic string in [32]. Supersymmetry is broken in these models by giving an expectation value to the holomorphic three-form of the Calabi-Yau threefold. More recently Giddings, Kachru and Polchinki [14] found the realization of this scenario in the context of $\mathcal{F}$-theory compactifications. In fact, the models described in [14] can be obtained from our models by a specific choice of eight-manifold that is an elliptic fibration over a threefold.

Let us mention briefly some concrete examples of compactifications of $\mathcal{M}$-theory on eight-manifolds that have appeared in the literature. All these examples involve non-compact internal manifolds and the relevant part of the $\mathcal{M}$-theory action (2.1) is S_0. A supersymmetric model in which $F = f_{(2,2)}$ where $f_{(2,2)}$ is primitive can be obtained by taking the internal space to be the eight-dimensional Stenzel metric [33]. A solution which breaks supersymmetry because the four-form is not primitive is given by the self-dual harmonic form on the complex line bundle over CP^3. It would certainly be interesting if concrete examples involving compact internal manifolds could be constructed since these models give rise to a finite three-dimensional Newton's constant. Some supersymmetric examples along these lines were constructed in [5].

5. Flux constraints and stabilization of moduli fields

Since the work of Dine and Seiberg it is well known that in string theory it is difficult to stabilize the moduli fields [1, 2]. Different shapes and sizes of the compactified dimensions lead to many physically inequivalent degenerate vacua. Recently Gukov, Vafa and Witten [4] found an interesting interpretation of the supersymmetry constraints (4.1) and (4.2). It was observed that the constraint

$$F_{(4,0)} = F_{(0,4)} = F_{(3,1)} = F_{(1,3)} = 0\,, \tag{5.1}$$

can be used to stabilize the complex structure moduli fields. This is because given a flux which satisfies the Dirac quantization condition the complex structure of the eight-manifold has to be adjusted in such a way that the constraint equations are satisfied. Furthermore, the condition

$$F_{(2,2)} \wedge J = 0\,, \tag{5.2}$$

can be used to fix many of the Kähler moduli of the internal manifold once the flux $F_{(2,2)}$ is used as an input. The radius of the eight-manifold (which is a Kähler modulus) cannot be determined though. The reason for this is that the equations are invariant under a rescaling with this parameter.

A corresponding interpretation of the constraints for type-IIB compactifications on six-manifolds [5] was found in [14]. Here a very nice derivation was made in terms of the supergravity potential. From this calculation it becomes clear how the discrete fluxes determine most of the moduli fields even if the vacua are not supersymmetric.

It would be interesting to derive the constraints found in this paper from a supergravity potential along the lines of [14]. The corresponding potential has been computed in [22].

6. Conclusion

In this paper we have found warped compactifications of $\mathcal{M}$-theory on (compact) eight-manifolds which generically are not supersymmetric and yet have a vanishing three-dimensional cosmological constant. Our calculation was based on a perturbative expansion in terms of the radius of the eight-manifold and took all the leading quantum corrections of $\mathcal{M}$-theory into account. Many of the moduli fields appearing in these compactifications can be stabilized using the constraints on the fluxes. These constraints have to be obeyed for the equations of motion and Bianchi identity to be satisfied.

It would certainly be interesting to extend the analysis performed in this paper to the next order in perturbation theory. It is conceivable that the compactification radius can be fixed in this way.

In order to do this calculation one first has to determine additional terms in the effective action of $\mathcal{M}$-theory. So for example to compute the next to leading order of the equation of motion for F_2 an additional term in the $\mathcal{M}$-theory action (2.1) becomes relevant

$$S_3 \propto \int \sqrt{-g} F^2 R^3\,. \tag{6.1}$$

This term has been considered previously in the literature [19, 34] but the coefficient of this interaction has not been determined so far.

However, it is also possible that non-perturbative effects of the form considered in [35, 36] stabilize the radius.

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